# Engineering Notes

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## Maximum Drag Reduction for a Flat Plate in Polymer Solution

PAUL S. GRANVILLE*
*Naval Ship Research and Development Center, Washington, D.C.*

### Nomenclature

$A$ = slope of logarithmic velocity law in natural logarithms
$\tilde{A}$ = slope of logarithmic interactive velocity law in natural logarithms
$\tilde{A}_1$ = slope of logarithmic interactive velocity law in common logarithms
$\tilde{\mathfrak{a}}$ = factor in logarithmic formula, Eq. (7)
$B_{1,0}$ = intercept of logarithmic velocity law for condition of no drag reduction
$\tilde{B}$ = intercept of logarithmic interactive velocity law
$\tilde{\mathfrak{B}}$ = factor in logarithmic formula, Eq. (7)
$c_1, c_2$ = linearizing factors in Eq. (6)
$C_F$ = over-all drag coefficient, Eq. (3)
$C_{F,0}$ = $C_F$ for condition of no drag reduction
$D_1, D_2$ = constants in Eq. (3)
$\mathfrak{D}$ = drag
$R_L$ = Reynolds number of flat plate
$S$ = surface area of flat plate
$u$ = velocity component parallel to flat plate
$u_\tau$ = shear velocity, $u_\tau = (\tau_w/\rho)^{1/2}$
$U$ = velocity of flat plate
$y$ = normal distance from flat plate
$\nu$ = kinematic viscosity of solution
$\rho$ = density of solution
$\tau_w$ = shearing stress at wall
$\%D.R.$ = percent drag reduction
ln = natural logarithm
log = common logarithm

THE development of the interactive layer concept[1] for turbulent shear flows with drag-reducing polymer solutions provides a method of predicting the maximum drag reduction for shear flows. A condition of maximum drag reduction develops if the shear layer is reduced to the laminar sublayer next to the wall and the interactive layer. A logarithmic law describes the interactive layer

$$u/u_\tau = \tilde{A}\ \ln u_\tau y/\nu + \tilde{B} \tag{1}$$

This law has the same form as that for fluids without drag reduction

$$u/u_\tau = A\ \ln u_\tau y/\nu + B_{1,0} \tag{2}$$

Hence, a prediction of maximum drag reduction may be readily made for flat plates with boundary layers with uniform concentration of polymer solution from the results of Ref. 2.

Equation (72) of Ref. 2 is rewritten in terms of $\tilde{A}$ and $\tilde{B}$ instead of $A$ and $B_{1,0}$ as

$$\ln R_L C_F = \frac{1}{\tilde{A}}(2/C_F)^{1/2} + 1 - \tilde{B}/\tilde{A} - \frac{B_2}{\tilde{A}} + \ln 2D_1 - \left(\frac{\tilde{A}}{2} + \frac{D_2}{D_1}\right)(C_F/2)^{1/2} \tag{3}$$

where $C_F$ is the over-all drag coefficient, $C_F \equiv \mathfrak{D}/(\frac{1}{2}\rho U^2 S)$ and $R_L$ is the Reynolds number, $R_L \equiv (UL)/\nu$. Here $B_2 = 0$ and then $D_1 = \tilde{A}$ and $D_2 = 2\tilde{A}^2$. Thus, there results in natural logarithms

$$\ln R_L C_F = \frac{1}{\tilde{A}}(2/C_F)^{1/2} + 1 - \tilde{B}/\tilde{A} + \ln 2\tilde{A} - \tfrac{5}{2}(C_F/2)^{1/2} \tag{4}$$

or in common logarithms

$$\log R_L C_F = \frac{1}{\tilde{A}_1}(2/C_F)^{1/2} + 1/2.3026 - \tilde{B}/\tilde{A}_1 + \log 2\tilde{A} - 5/2(2.3026)(C_F/2)^{1/2} \tag{5}$$

where $A_1 = 2.3026A$.

If $C_F^{1/2}$ is linearized with respect to $C_F^{-1/2}$ over the range of Reynolds number of interest or

$$C_F^{1/2} = c_1 + c_2 C_F^{-1/2} \tag{6}$$

where $c_1$ and $c_2$ are constants, then the usual flat-plate form is achieved for the drag coefficient for maximum drag reduction

$$\log R_L C_F = \tilde{\mathfrak{a}} C_F^{-1/2} + \tilde{\mathfrak{B}} \tag{7}$$

where

$$\tilde{\mathfrak{a}} = 2^{1/2}/\tilde{A}_1 - 5c_2/2(2.3026)2^{1/2}$$

and

$$\tilde{\mathfrak{B}} = 1/2.3026 - \tilde{B}/\tilde{A} + \log 2\tilde{A} - 5c_1/2(2.3026)2^{1/2}$$

Values of $\tilde{A} = 11.7$ and $\tilde{B} = -17.0$ are given by Virk et al.[1] Figure 1 shows the corresponding plot. For comparison a power-law relation derived by Giles[3]

$$C_F = 0.315/R_L^{0.302} \tag{8}$$

is also plotted. Also plotted are the laminar drag coefficient

$$C_F = 1.328/R_L^{1/2} \tag{9}$$

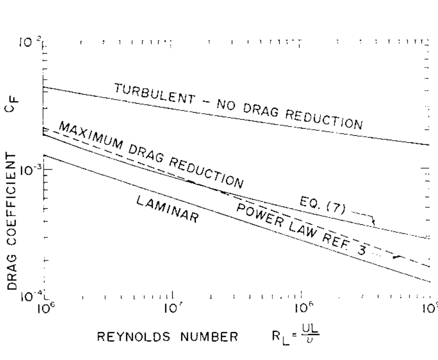


**Fig. 1 Drag coefficient for maximum drag reduction of flat plates.**

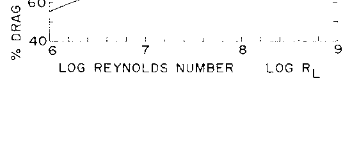


**Fig. 2 Percent maximum drag reduction for flat plates.**

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* Research Specialist, Ship Performance Department.

and the turbulent Schoenherr law for no drag reduction $C_{F,0}$,

$$\log R_L C_{F,0} = 0.242 C_{F,0}^{-1/2} \quad (10)$$

The percentage of drag reduction, $\%D.R.$, is also plotted in Fig. 2

$$\%D.R. = (1 - C_F/C_{F,0})100\% \quad (11)$$

where $C_F$ is the drag coefficient for polymer solutions and $C_{F,0}$ is the drag coefficient for no drag reduction at the same Reynolds number. The results are most favorable.

It should be noted that the maximum drag reduction is predicted on the basis of a theoretical model. In practice high-shear stresses will probably mechanically degrade the polymer molecules and diminish the drag reduction so that the maximum drag reduction may not be attained. The actual friction line lies then between the Schoenherr line of no drag reduction and the line of maximum reduction. This line may be determined by the method of Ref. 2.


### References

[1] Virk, P. S., Mickley, H. S., and Smith, K. A., "The Ultimate Asymptote and Mean Flow Structure in Toms' Phenomenon," *Transactions of ASME, Journal of Applied Mechanics*, Vol. 37, Ser. E, No. 2, June 1970, pp. 488–493.

[2] Granville, P. S., "Frictional Resistance and Velocity Similarity Laws of Drag-Reducing Dilute Polymer Solutions," *Journal of Ship Research*, Vol. 12, No. 3, Sept. 1968, pp. 201–212.

[3] Giles, W. B., "Similarity Laws of Friction-Reduced Flows," *Journal of Hydronautics*, Vol. 2, No. 1, Jan. 1968, pp. 34–40.


# Added Mass of a Circular Cylinder in Contact with a Rigid Boundary


C. J. Garrison*

*Naval Postgraduate School, Monterey, Calif.*


## Introduction

THE calculation of the forces exerted on an object placed in an accelerating flowfield is a fluid mechanics problem of considerable practical importance. For example, in the calculation of wave forces acting on submerged bodies it is necessary, in general, to know both the added mass and drag coefficient for the particular shape in question. The wave force is generally considered to consist of two parts, drag and inertia. However, if the object involved is large in comparison to the amplitude of the fluid motion, separation does not occur and the resulting wave induced oscillatory flow may be considered an unseparated, potential flow about a fixed object. In this case the drag coefficient is necessarily zero while the added mass coefficient is dependent on the shape of the body in question and its proximity to any rigid boundaries. In problems of this type, the potential flow value of the added mass coefficient is of considerable practical importance since it is the basic shape dependent factor required for calculating forces.

One configuration of particular importance in view of its prevalence in engineering structures is the circular cylinder. When a circular cylinder is immersed in a fluid of infinite extent its added mass coefficient is easily calculated and has the well-known value of $C_m = 1.0$. (The inertia coefficient is defined as $C_I = 1.0 + C_m$ and accordingly has the value of 2.0 for the circular cylinder.) However, when the circular cylinder is placed near a solid boundary, as shown in Fig. 1, and the fluid accelerated parallel to the boundary, the added mass is increased somewhat due to the presence of the rigid boundary. This particular configuration consisting of a circular cylinder in contact with a rigid boundary has obvious practical significance in application to wave forces exerted on bottom mounted structures such as submerged pipelines and oil storage tanks. In these cases it is of interest to know the effect of the proximity of the bottom on the added mass coefficient.


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* Assistant Professor, Department of Mechanical Engineering.

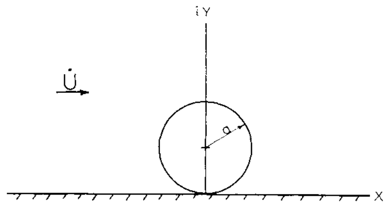


**Fig. 1 Definition sketch.**

The problem under consideration was considered previously by Dalton and Helfinstine[1] as the special limiting case of two circular cylinders approaching touching. Their approximate method, based on the use of images, involved repeated application of Milne-Thomson's circle theorem in order to generate potential flow past two cylinders in close proximity. Half of this configuration corresponds to the present situation of a cylinder touching a plane boundary in view of the symmetry involved. However, as touching became imminent their solution broke down locally on account of the singularity at the point of contact and, therefore, the accuracy of the value of $C_m$ so obtained is questionable. This is unfortunate since the just touching case is the most important with respect to practical application.

In the present Note the case of a circular cylinder in contact with a plane boundary is treated. The complex potential for this case is expressed in closed form and is applied to calculate the net force in the direction of fluid acceleration. From the inertia coefficient so obtained, the added mass coefficient for the circular cylinder in contact with a plane boundary is calculated in closed form without recourse to approximate methods.

## Inertial Force

It is well known that the drag force exerted on an object in a potential flow (without circulation) is zero while the added mass or inertial force is dependent upon the shape of the object and proximity to solid boundaries. In order to evaluate this inertia force it is necessary to determine the pressure distribution around the cylinder. Such information can be obtained from consideration of the unsteady form of Bernoulli's equation for an incompressible fluid,

$$P/\rho + q^2/2 + gh + \partial\phi/\partial t = \Pi(t) \quad (1)$$

where $P$ denotes the pressure at any general point within the fluid, $q$ the local velocity, $h$ the elevation above some arbitrary datum, $\rho$ the fluid density, $g$ the acceleration of gravity, $\phi$ the velocity potential and $\Pi(t)$ some function of time only.

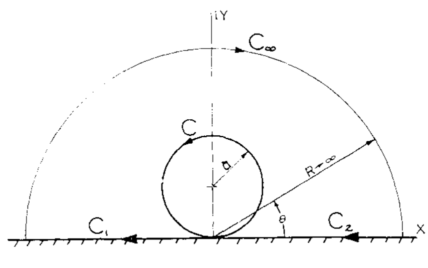


**Fig. 2 Path of integration.**